HEAT AND MASS TRANSFER IN POROUS MEDIA

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A brief survey of works concerned with transfer processes in porous media is given.

1. METHOD OF AVERAGES

One of the main problems encountered in describing heat and mass transfer processes in porous media (as in heterogeneous media in general) is to obtain a system of corresponding macroscopic equations for average quantities. In the phenomenological theory of heterogeneous media such a system is derived using the laws of conservation of mass, momentum, and energy for each phase. However, to close the system, it is necessary to adopt additional assumptions when determining quantities describing interphase interactions.

The method of averages makes it possible both to derive the above equations in a more rigorous way and to obtain expressions for the interphase interaction intensity, stress tensor in phases, energy flux, etc. in terms of the instantaneous values of microparameters in the phases (including pulsation components). In order to close the equations, these quantities must then be expressed in terms of the macroscopic (average) parameters and their derivatives.

Behind the method of averages, described in detail in the monograph [1] and in [2, 3], lies the following. Let us consider a heterogeneous medium consisting of two phases. Choose [1] the volume element $dV = dV_1 + dV_2$ containing the interface S_{12}. A dimension of this volume is, on the one hand, much less than the characteristic dimension of the body but, on the other hand, it considerably exceeds the dimensions of microinhomogeneities (pores). Next, introduce the volume concentration of the phases $\varepsilon_i = dV_i/dV$ and the relative interface $s_{12} = S_{12}/dV$. The macroscopic quantities are introduced by performing averaging over the phase volumes dV_i and interfaces dS_{12} as follows:

$$\langle \varphi_i' \rangle_i = \frac{1}{dV_i} \int_{dV_i} \varphi_i' \, d'V, \quad \langle \varphi_i' \rangle_{12} = \frac{1}{dS_{12}} \int_{dS_{12}} \varphi_i' \, d'S.$$

Here

$$\langle \varphi' \rangle_V \equiv \langle \varphi' \rangle = \varepsilon_1 \langle \varphi_1' \rangle_1 + \varepsilon_2 \langle \varphi_2' \rangle_2$$

(the primed quantities indicate instantaneous values that are averages in the microvolumes $d'V$).

It should be noted that in averaging over the phases (i.e., over a part of the volume dV_i), unlike in averaging over the entire volume, the average of the time t derivative can differ from the time derivative of the corresponding averaged function ($\langle \partial \varphi_i'/\partial t \rangle_i \neq \partial \langle \varphi_i' \rangle_i/\partial t$) [1-3]. A similar situation also occurs in differentiating with respect to the space coordinate. We give a formula that relates the space derivative of the phase-averaged function to the averaged space derivative of the instantaneous values of the corresponding functions [1-3]:

$$\varepsilon_i \langle \nabla^k \varphi_i' \rangle_i = \nabla^k \varepsilon_i \langle \varphi_i' \rangle_i + s_{12} \langle \varphi_i' n'^k \rangle_{12} = \nabla^k \varepsilon_i \langle \varphi_i' \rangle_i + \frac{1}{dV} \int_{dS_{12}} \varphi_i' \, n_i'^k \, d'S,$$

where n_i' is the outer (with respect to the i-th phase) normal to the interface s_{ij} ($i, j = 1, 2$), and the superscript k pertains to the corresponding coordinate.

Now it is necessary to apply averaging to the equations of conservation of mass, momentum, and energy, which describe micromotion in the phases, by integrating them over the microvolume dV_i. In particular, the equation of conservation of mass of the i-th phase acquires the form

$$\frac{\partial \bar{\rho}_i \, \varepsilon_i}{\partial t} + \mathrm{div}\,(\bar{\rho}_i \, \varepsilon_i \, \mathbf{v}_i) = J_{ji},$$ (1)

where $\bar{\rho}_i = \langle \rho_i' \rangle_i = \langle \rho_i' \rangle_V / \varepsilon_i$ is the actual density [1] (the mass of the i-th phase per unit volume of the i-th phase), $\mathbf{v}_i$ is the vector of the mean mass velocity $(\bar{\rho}_i \mathbf{v}_i = \langle \rho_i' \mathbf{v}_i' \rangle_i)$, and the quantities J_{ij} determine the averaged intensity of phase transitions (if any) on the interface,

$$J_{21} = s_{12} \langle \bar{\rho}_1' \, (v_1'^n - N'^n) \rangle_{12} = \frac{1}{dV} \int_{S_{12}} \bar{\rho}_1' \, (v_1'^n - N'^n) \, d'S \, ; \quad J_{21} = - J_{12} \, ,$$

where N'^n is the velocity component of interface displacement.

Similarly, we can represent the equations of conservation of momentum of the i-th phase for the averages (with account for the tensor of pulsation stresses) and the equation for the internal energy of the i-th phase (or for the total specific energy of the i-th phase, consisting of the internal energy, kinetic energy of macromotion, and kinetic energy of pulsations) [1].

Consider a saturated porous medium consisting of a solid phase (phase 2) whose pores are filled with a liquid or a gas (phase 1). In a porous medium with an immovable solid phase ($v_2 = 0$), the averaged tensor of viscous stresses in phase 1 is neglected (i.e., the viscosity is taken into account only in the force of interphase interaction). Moreover, the inertia forces, momentum transfer by pulsations, and kinetic energy of pulsations are also considered to be negligible. With the above assumptions taken into consideration, from the equation of conservation of momentum an expression follows that corresponds to the linear Darcy law relating the rate of liquid (gas) filtration $\mathbf{w} \equiv \langle v_1' \rangle = \varepsilon \mathbf{v}_1$ to the pressure gradient ∇p_1 in a saturated isotropic porous medium:

$$\mathbf{w} = \varepsilon \, \mathbf{v}_1 = - \frac{k_0}{\mu} \nabla p_1 \, ,$$ (2)

where ε is the porosity; k_0 is the penetration factor (with the dimensionality of area); μ is the dynamic viscosity. In the literature, various empirical expressions are known for k_0. In particular, according to the Kárman–Kozeni formula [4], $k_0 = \varepsilon^3 / 5 S^2$ (S is the specific surface of the pores). For a model porous body consisting of homogeneous solid spheres with diameter d_s, $k_0 = \varepsilon^2 d_s^2 / 180 (1 - \varepsilon)^2$. The square law of filtration [4] is also used:

$$\nabla p_1 = - \frac{\mu}{k_0} \mathbf{w} - \beta \frac{\bar{\rho}_1 \, w}{\sqrt{k_0}} \mathbf{w} \, ,$$ (3)

where β is an additional characteristic of the porous medium. The quadratic term in (3) plays an important role, for instance, in media consisting of coarse particles. A deviation from the Darcy law at low filtration rates is observed in colloidal capillary-porous bodies and in the case of filtration of rheological fluids [4].

Saturation θ of a porous body with such a fluid β is determined by the relative part of the void volume occupied by this fluid ($\theta = \varepsilon_\beta / \varepsilon$). For saturated porous media ($\theta = 1$), using the continuity equation (1) (in the absence of phase transitions $J_{ij} = 0$) and the Darcy law (2) and omitting the subscript 1 for $\bar{\rho}$, p, we arrive at

$$\frac{\partial \varepsilon \bar{\rho}}{\partial t} = \mathrm{div} \left(\bar{\rho} \, \frac{k_0}{\mu} \nabla p \right) .$$

Proceeding from the assumption of weak compressibility for the liquid and the porous medium, i.e., considering the relative changes in $\bar{\rho}$ and ε to be small, from the latter equation we can write an equation for an elastic filtration regime [5]:

$$\frac{\partial p}{\partial t} = \kappa \nabla^2 p \equiv \kappa \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right),$$

which coincides with the classical heat conduction equation (κ is the piezoconductivity). To describe gas filtration, the compressibility of which is considerably higher than that of the porous medium, the following equation is obtained for the isothermal case [5]:

$$\frac{\partial p}{\partial t} = \frac{k_0}{2\mu\varepsilon} \nabla^2 p^2.$$

Rigorous allowance for elastic deformations of the solid phase will be discussed below.

2. UNSATURATED POROUS MEDIA AND SURFACE PHENOMENA

In investigations of heat and mass transfer processes with phase changes in porous materials unsaturated porous media attract great interest. The phase interaction in a body complicates transfer processes considerably, thus leading to interrelated heat and mass transfer. This is of particular importance for moisture flow investigations in soils, development of drying technologies for moist materials, and impregnation of the latter.

In unsaturated media, surface phenomena play an important role. Capillary pressure is an additional pressure of the surface layer of the liquid which is caused by surface curvature. In thin layers of liquids (with a thickness of about 10^{-5} cm) wedging pressure develops [6]. In this case, the hydrostatic pressure in a thin film differs from the pressure in the adjacent volume phase, which is related to the forces of molecular interaction between the liquid in the thin layer and the adjacent phase.

The wedging pressure $\Pi(h)$ depends on the film thickness h and is equal to the difference between the pressure on the film surface and the pressure in the volume phase. If the film thickness $h(x)$ changes over the length, then a gradient of the wedging pressure develops and, as a consequence, liquid flow. Therefore, in calculations of the rate of liquid evaporation from a cylindrical capillary of radius r the expression for the total vapor flow $\overline{Q}$ (g/sec) can be written as a sum of the volume Q_V and surface Q_s flows [6]:

$$\overline{Q} = \pi r^2 Q_V + 2\pi r Q_s = -\frac{\pi r^2 DM}{RT}\frac{dp}{dx} + \frac{2\pi r \rho_f h^3}{3\mu}\frac{d\Pi}{dx}, \tag{4}$$

where D is the vapor diffusion coefficient; M is the molecular weight; $\overline{R}$ is the universal gas constant. The last term in (4) describes the substance flow in the adsorbed film between the meniscus and the capillary opening. The contribution of the film flow to the total mass flow in evaporation from capillaries increases with the relative vapor pressure in the surrounding medium, i.e., with increasing $\varphi = p_V/p_e$ (p_V is the vapor partial pressure in the surrounding medium, p_e is the saturated-vapor pressure). In particular, for water at $\varphi \le 0.75$ the influence of the film flow can be neglected.

In [6], the authors describe investigations of the surface forces and the different wedging-pressure components as well as various phenomena of mass transfer caused by special properties of thin layers of liquids. Such phenomena include, in particular, capillary osmosis, i.e., the occurrence of a liquid flow in the presence of a concentration gradient of a dissolved substance along a capillary, and thermocrystallization-induced flow of thin nonfreezing interlayers between ice and a solid surface (the existence of which in soils and grounds is related to surface forces) due to temperature and pressure gradients.

3. INTERRELATED HEAT AND MASS TRANSFER INVOLVING PHASE CHANGES

Using methods of the thermodynamics of irreversible processes, A. V. Luikov has derived a system of equations for various phenomena and their interrelations [7]. Here, the corresponding potentials and macroscopic transfer coefficients are also determined. In particular, the interrelated heat and moisture transfer in capillary-

porous bodies with allowance for phase changes, provided that the total change in the specific moisture content of a body u is attributable to moisture transfer and liquid-into-vapor phase transformation, is described in the one-dimensional case by the following system of equations:

$$\frac{\partial T}{\partial t} = \frac{1}{c_p \rho_0} \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\psi Q}{c_p} \frac{\partial u}{\partial t},$$ (5)

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(\chi' \frac{\partial u}{\partial x} + \chi' \delta' \frac{\partial T}{\partial x} \right).$$ (6)

In the general case, the transfer coefficients depend on the moisture content u and the temperature T. Here $\chi' = \lambda'/\rho_0 c'$ is the vapor diffusion coefficient; ρ_0 is the density of the dry body; c' is the specific isothermal mass; λ is the thermal conductivity; c_p is the specific heat; δ' is the thermal-gradient coefficient, which includes the ratio of the thermal-diffusion and moisture-diffusion coefficients as one of the summands; ψ is a criterion of phase changes that characterizes the ratio of the change in the moisture content due to evaporation to the total change in the moisture content; Q is the heat of phase transformation. The criterion ψ is most often treated as a continuous function of the coordinates (or the moisture content). In the presence of deepening of the evaporation zone, the system of equations (5) and (6) is considered for each zone, while $\psi(u)$ is represented in the form of a discontinuous function.

In the case of high-rate drying processes inside a moist material, a total-pressure gradient exists since the pressure drop of moist air caused by liquid evaporation does not relax instantaneously. In this case, the system of equations (5) and (6) changes, namely, in each of these equations a term proportional to Δp appears on the right-hand side and an equation for p (diffusional-filtrational moisture transfer [7]) is appended.

Allowance for the finite velocity of substance propagation in the context of the linear theory of interrelated heat and moisture transfer considered leads to the system of equations [4]

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c_p \rho_0} \frac{\partial^2 T}{\partial x^2} + \frac{\psi Q}{c_p} \frac{\partial u}{\partial t},$$

$$\frac{\partial u}{\partial t} + \tau' \frac{\partial^2 u}{\partial t^2} = \chi' \frac{\partial^2 u}{\partial x^2} + \chi' \delta' \frac{\partial^2 T}{\partial x^2}.$$

The last expression is a hyperbolic equation, while the energy equation is an ordinary parabolic equation, since the relaxation time of the heat flux is considerably smaller than that of the mass flow τ'. It should be noted that the effect of the finite velocity of moisture propagation can be also described by a nonlinear convective-diffusion equation for unsaturated media.

The system of equations of A. V. Luikov is widely used to describe drying processes. In the period when the drying rate decreases, the evaporation zone is observed to deepen inside the body. If vapor removal is so vigorous that the mechanism of capillary transfer does not provide replenishment of the dried pores with liquid, then a front is formed that moves into the body (a high-rate drying regime [8]). The mathematical model of such a process is based on a Stefan-type problem for whose correct formulation an additional expression is needed in order to relate the velocity of the evaporation front to the temperature on it. In some cases, for model porous media such a relation can be obtained from consideration of the kinetics of mass transfer in individual capillaries [9]. In the general case, evaporation proceeds not only on the deepened surface but in some zone as well. This is attributable, in particular, to various forms of binding of moisture with the material.

4. MULTIPHASE FILTRATION

The recognized merits of the theory of A. V. Luikov are its generality and simplicity, while its drawbacks lie in the impossibility of finding the phase distribution in a porous body and of evaluating the role of different

mechanisms of moisture transfer. Therefore, multiphase-filtration theory is often used, in which the basic assumption is the independence of motion of individual phases. In particular, in the case of filtration of a two-phase mixture of a liquid and its vapor it is assumed that the Darcy law (or a generalization of it) is valid for each of the phases:

$$w_g = \varepsilon \theta_g \, v_g = - \frac{k_0 k_g}{\mu_g} \, \nabla p_g \, , \tag{7}$$

$$w_f = \varepsilon \theta_f \, v_f = - \frac{k_0 k_f}{\mu_f} \, \nabla p_f \, , \tag{8}$$

where θ_g, θ_f are the saturation of a porous body with gas and liquid, respectively; k_g, k_f are the relative penetrabilities of the gas and liquid phases, which are functions of the saturation [2, 5]. The pressures in the gas and liquid phases in an actual porous body are related through the capillary pressure p_σ by the expression

$$p_g - p_f = p_\sigma = 2\sigma \cos \varphi \, \sqrt{\left(\frac{\varepsilon}{k_0} \right)} \, f(\theta_f) \, , \tag{9}$$

where φ is the wetting angle; $f(\theta_f)$ is the Leverett function [4, 5, 10].

With neglect of the diffusion process in the assumption of rather intense evaporation, the continuity equation for the gas phase in the one-dimensional case acquires the form

$$\varepsilon \, \frac{\partial \theta_g \bar{\rho}_g}{\partial t} + \frac{\partial (\bar{\rho}_g w_g)}{\partial x} = J_{fg} \, , \tag{10}$$

where the expression for the evaporation rate can be approximately written as [11]

$$J_{fg} = \frac{p_e(T) \exp \left\{ - \dfrac{p_\sigma M}{\overline{RT\rho_1}} \right\} - p_g}{\sqrt{2\pi \overline{RT}/M}} - \varepsilon \, \frac{\partial \theta_g}{\partial x} \, .$$

The continuity equation for the liquid phase is similar to (10); here $J_{gf} = -J_{fg}$.

Now we write the energy equations under the assumption that the temperature of the solid phase (skeleton) T_s differs from that of the gaseous and liquid phases T:

$$(1 - \varepsilon) \bar{\rho}_s \, c_{ps} \, \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x} \left[\lambda_s (1 - \varepsilon) \frac{\partial T_s}{\partial x} \right] - \alpha_V (T_s - T) \, , \tag{11}$$

$$\varepsilon \, [\theta_g \bar{\rho}_g \, c_{pg} + \theta_f \bar{\rho}_f \, c_{pf}] \frac{\partial T}{\partial t} + [\bar{\rho}_g \, w_g \, c_{pg} + \bar{\rho}_f \, w_f \, c_{pf}] \frac{\partial T}{\partial x} - J_{fg} Q = \frac{\partial}{\partial x} \left[\lambda_{f\,eff} \frac{\partial T}{\partial x} \right] + \alpha_V (T_s - T) \, . \tag{12}$$

Here Q is the specific heat of vaporization, and α_V is the volume heat transfer coefficient.

In this model an important problem is determination of the effective thermal conductivity of the heat carrier $\lambda_{f\,eff}$ and the internal heat transfer coefficient α_V. Many works devoted to these aspects have been published in recent years. Thus, in [10, 12], for a porous body consisting of spherical solid particles of diameter d_s it is suggested that use the following expressions for $\lambda_{f\,eff}$ and α_V be used:

$$(\lambda_{f\,eff})_x = \varepsilon \, \lambda_f + 0.5 \lambda_f \, \mathrm{Pr} \, \mathrm{Re}_d$$

(in the two-dimensional case, in the expression for $(\lambda_{f\,eff})_y$ the coefficient 0.1 enters the second term on the right-hand side),

$$Nu = 2 + 1.1\, Pr^{1/3}\, Re_d^{0.6},$$

where $Nu = \alpha d_s/\lambda_f$ is the Nusselt number,

$$S = 6\,(1 - \varepsilon)/d_s, \quad Re_d = \frac{\overline{\rho}_f\, v_{fx}\, d_s}{\mu_f}, \quad Pr = \mu_f\, c_{pf}/\lambda_f.$$

It is pertinent to note that at high filtration rates, i.e., at high Peclet numbers ($Pe = Re_d Pr \gg 1$), when the conductive heat transfer is often neglected, dispersion effects caused by irregularity of the velocity field due to the random structure of the porous medium play an important role and lead to a considerable increase in the effective thermal conductivity [13]. In this case, $\lambda_{f\,eff}$ becomes an anisotropic quantity, since the dispersion effects are most pronounced in the direction of filtration.

In [14], on the basis of an analysis of momentum transfer on a porous medium—homogeneous-liquid flow interface, two conjugation boundary conditions are obtained for the flows in these media. One of them is equality of the average velocities at the interface, while the other is the relationship between the transverse derivatives of the average velocities and the average velocity in the porous medium.

Based on the system of equations for two-phase filtration, mathematical models for processes of drying [2, 8] and impregnation and forced replacement of a liquid [5] have been developed.

5. CONVECTIVE DIFFUSION IN POROUS BODIES

The irregularity of the velocity fields in actual porous bodies is responsible for both the increase in the effective thermal conductivity and the dispersion of a foreign dissolved substance or tagged liquid particles in the course of liquid filtration [15-17]. In these cases, the total mass transfer is described by an equation of convective diffusion with an effective diffusion (dispersion) coefficient dependent on the mean velocity.

In the simplest case, when instead of the porous medium some equivalent capillary tube of radius r is used, we have [4, 15]

$$\frac{\partial \tilde{c}}{\partial t} + w\frac{\partial \tilde{c}}{\partial x} = D_{eff}\frac{\partial^2 \tilde{c}}{\partial x^2},$$

where $D_{eff} = D + r^2 w^2/48D$; $\tilde{c}$ is the mean impurity concentration over the tube cross section; D is the molecular-diffusion coefficient; w is the filtration rate. However, for complex porous media such an approach is rather crude.

In [15, 16], the following approach is suggested to describe convective diffusion. In averaging the local equation of convective diffusion in an individual pore (but with allowance for fluctuations in the velocity and concentration fields), the assumption is made that the average over the product of the velocity and concentration fluctuations is an additional pulsation flux of the substance proportional to the gradient of the mean (over the pore space) concentration. The proportionality factor is the tensor of convective diffusion, dependent on the average-velocity vector $\overline{v}$:

$$D_{\alpha\beta} = (\lambda_1 - \lambda_2)\frac{\overline{v}_\alpha\,\overline{v}_\beta}{|\overline{v}|} + \lambda_2\,|\overline{v}|\,\delta_{\alpha\beta},$$

where λ_1 and λ_2 are, respectively, the longitudinal and transverse scattering parameters of the porous medium, having the dimension of length; the subscripts α, β pertain to the Cartesian coordinates; $\delta_{\alpha\beta}$ is the unit tensor. In [16] it is noted that $\lambda_2/\lambda_1 \approx 0.1$, i.e., the longitudinal dispersion is more pronounced than the transverse one, and $\lambda_1 \approx 10^2\sqrt{K_0}$. In the general case, the parameters λ_1 and λ_2 depend on the diffusional Peclet number $Pe = \overline{v}d_s/D$. If we assume that the filtation rate direction coincides with the x_1 axis, then the convective-diffusion tensor $D_{\alpha\beta}$ can be replaced by the longitudinal $D_{\parallel} = \lambda_1\overline{v}$ and transverse $D_{\perp} = \lambda_2\overline{v}$ diffusion coefficients. Then the diffusion equation acquires the form [15]
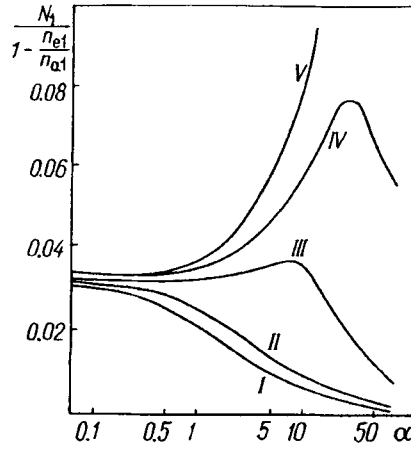
Fig. 1. $N_1/(1 - \eta_{e1}/\eta_{a1})$ versus α for $l = 30$: I) $\eta_{a1}/\eta_{a2} = 0.1$, II) 1, III) 10, IV) 100, V) one-component flow.

$$\varepsilon \frac{\partial \bar{c}}{\partial t} + w \frac{\partial \bar{c}}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\lambda_1 \ w \frac{\partial \bar{c}}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\lambda_2 \ w \frac{\partial \bar{c}}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\lambda_3 \ w \frac{\partial \bar{c}}{\partial x_3}\right),$$

where $w = \varepsilon \bar{v}$ is the filtration rate, and $\lambda_2 = \lambda_3$.

6. KINETIC METHODS FOR DESCRIBING TRANSFER PROCESSES IN POROUS MEDIA

The choice of kinetic methods to describe gas flows in porous media depends on the model of a porous body adopted. In particular, the models of nonintersecting capillaries and a "dust-laden" gas [18-20] allow use of methods of the kinetic theory of gases.

In the case of the capillary model, calculations are based on expressions obtained in gas flow investigations in cylindrical channels for various Knudsen numbers (Kn is equal to the ratio of the mean free path of the molecules to the capillary radius). In [8, 20], various interpolation formulas are given for a gas flow (for both one-component and gaseous-mixture flows) through a capillary that provide exact expressions in the limiting cases of the Knudsen (Kn → ∞) and continuum (Kn → 0) regimes as well as satisfactory agreement with experimental data in the transient regime (Kn ≈ 1), reached by means of fitting parameters.

Similar interpolation formulas have also been obtained to describe diffusion processes in a porous medium [8], but here, as a result of averaging, effective diffusion coefficients (in the absence of convection) emerge.

It should be noted that such expressions do not allow for the features related to the presence of phase or chemical changes, movable adsorbed layers in capillaries, and a deepened evaporation boundary in a porous body. These factors have been taken into account in [9] by solving model kinetic equations describing the flow of a binary vapor-gas mixture in a cylindrical capillary of radius r in which the filler (component 1) evaporates at the depth L. Component 2 is a foreign gas (e.g., air) whose density at the channel inlet is n_{a2}, and $n_{a1} < n_{e1}$ (n_{e1} is the saturated-vapor density). Note that in the present problem the partial-pressure gradients are not specified (unlike the Poiseuille problem) but are determined in solving the problem. In Fig. 1 the reduced vapor flow rate N_1 is represented as a function of the rarefaction parameter $\alpha \sim 1/Kn$ for different density ratios n_{a1}/n_{a2}. It is shown, in particular, that a neutral gas, added even in a small amount, exerts a pronounced influence on both the magnitude and character of the N_1 vs. α curve (as compared to the limiting case of a one-component flow). Consideration is given to the domains of applicability of the known phenomenological relations, namely, Fick's law, the expression for a Stefan flow, and the Poisseuille formula.

A similar kinetic problem of mass transfer in the case of one-component vapor filtration and the presence of a deepened evaporation zone has also been considered [21] for the "dusty gas" model, i.e., for a highly porous body modeled by a homogeneous system of randomly distributed spherical particles (Fig. 2). Simple expressions are derived for the evaporation rate, penetrability factor, and density jump (outlet resistance) at the gas−porous-
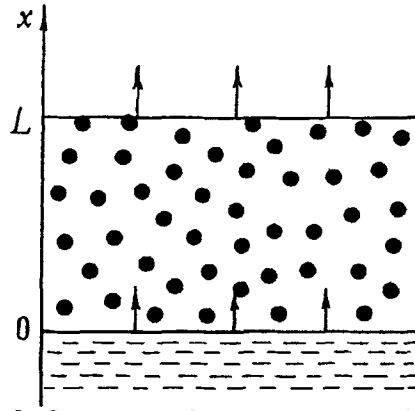
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Fig. 2. Geometry of a highly porous layer.

medium interface as a function of the Knudsen number (here Kn is equal to the ratio of the mean free path to the radius of a coarse particle r). In particular, the expression for the dimensionless evaporation rate u_L is as follows:

$$u_L = - \frac{\varepsilon^2 v_L}{\pi^{1/2} (4 - 2\varepsilon - \varepsilon^2 + A)} ,$$

where ε is the porosity; $v_L = (n_L - n_e)/n_e$,

$$A = \frac{9(1-\varepsilon)}{\pi} \frac{L}{r} \mathrm{Kn}\, B\,(\mathrm{Kn}) ; \quad B = \frac{15 - 3\,\mathrm{Kn} + 3\,(8 + \pi)\,\mathrm{Kn}^2}{15 + 12\,\mathrm{Kn} + 18\,\mathrm{Kn}^2 + 54\,\mathrm{Kn}^3} .$$

The plot of u_L versus $1/\mathrm{Kn}$ is a curve that at first increases very slowly from the free-molecule value. Then starting approximately from $\mathrm{Kn} = 1$ the parameter u_L increases more sharply, thus turning into the linear dependence $u_L \sim 1/\mathrm{Kn}$ typical for a viscous gas flow. Finally, with further decrease in the Kn number, u_L reaches a constant value corresponding to evaporation from an open surface. A comparison of the results obtained and experimental data has shown that in the region of low Kn it is necessary to introduce a correction factor that depends on the porosity and allows for flow "constriction" in the porous medium.

7. RADIATIVE TRANSFER

When porous materials are exposed to radiation, the researcher is faced with the important problem of simultaneously describing radiation transfer and absorption, on the one hand, and the temperature field in the porous body with the internal heat source formed, on the other hand. In particular, these aspects are investigated in [22] in relation to the problem of the drying of capillary-porous materials (foodstuff) by infrared radiation. The basic challenge in such problems is determination of the effective optical properties of a volume element in the porous (dispersed) medium. For instance, the absorption and scattering coefficients are often calculated for an individual scattering and then the result obtained is multiplied by the number of scatterers per unit volume. In [23], the effective absorption and scattering coefficients of a bed consisting of randomly packed spheres are determined using two-flow equations and the Monte Carlo method.

However, it is not always advisable to consider inhomogeneous scattering and absorbing media as pseudohomogeneous media with volume-averaged optical properties. In cases where particular models of porous (or dispersed) media are used, other methods can also be employed.

In [24], a mathematical model is proposed for the effect of radiation fluxes in vacuum on porous bodies with a capillary structure with allowance for radiation transfer (channeling) in pores with nontransparent walls. It is shown that in the case of low emissivities of the material of the capillary walls ε_r, it is important to know the distribution of the heat source over depth, since with decreasing ε_r the difference in the energies absorbed by the

body in its volume and over its surface increases. This follows from the formula for the ratio of the total absorbed radiation energy κ to the incident radiation flux j_0:

$$\kappa/j_0 = \varepsilon_r + \varepsilon \left[\frac{\kappa^* + j_1}{j_0} - \varepsilon_r \right] ,$$

where κ^*/j_0, j_1/j_0 are the portions of radiation flux absorbed by the lateral surface of the capillaries and their bottom, respectively. The quantity in square brackets tends to zero as $\varepsilon_r \to 1$.

This feature is most pronounced in the case of pulsed-periodic irradiation. The model mentioned is used in [25] to describe heat and mass transfer in composite materials such as glass-fiber plastic, exposed to radiation fluxes, with allowance for their loosening (the formation of a porous layer).

In practice, porous bodies with a globular structure are often used. In such cases, the model of randomly arranged spheres is more adequate. In [26], a survey is made of the state-of-the-art of the problem of radiative heat transfer in packed and fluidized beds that are multiphase systems consisting of solid particles (spheres) and a gas. The choice of the methods for calculating the scattering and absorption characteristics of a multiparticle system depends on a mode of radiation scattering by the system. In that work the author presents (for a disordered distribution of particles) a diagram of independent and collective scattering by particles in relation to the volume portion of particles and the parameter $\alpha = \pi d_s/\lambda_r$ or the gap between the particles L_d (d_s is the particle diameter, λ_r is the radiation wavelength). It is shown that the line $L_d/\lambda_r = 0.5$ separating the regimes of dependent and independent scattering corresponds to a 5% deviation from the Mie theory of independent scattering.

Radiation transfer in a layer of a highly porous body with random arrangement of nontransparent particles of the skeleton can be described by an integral transfer equation obtained in the approximation of the mean free path of photons [27].

Based on simultaneous solution of the problems of radiation transfer and unsteady heat conduction in a highly porous layer, it is shown that the power density distribution of the absorbed energy can differ considerably from an exponential dependence of the Bouguer type. In particular, for a collimated flux of external radiation the mentioned distribution can have a maximum inside the body or be a monotonically decreasing function of the coordinate, depending on the porosity, layer thickness, emissivity, and heating time.

Use of the model described in [27] allows evaluation of the effective emissive power of an isothermal highly porous (dispersed) layer, which tends to an asymptotic value with increase in the layer thickness [28]:

$$\overline{\varepsilon}_{r\infty} = (1 - \varepsilon) \varepsilon_r + \frac{2\varepsilon \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} ,$$

where ε is the porosity, ε_r is the emissivity of a particle surface. In the nonisothermal case, an approximate analytical expression is obtained for the radiation flux density at the layer outlet.

Numerous applied problems are related to the interaction between radiation and other mechanisms of heat transfer in porous media. In the case of conductive-radiative heat transfer the radiation transfer equation must be solved simultaneously with the equation of the energy in a porous body. For this, iteration methods are usually employed. The situation becomes even more complicated if convective heat transfer is also involved. A survey of such works is made in [26].

Recently impetus has been given to investigations of radiative-convective heat transfer in porous bodies aimed at implementation of heat recovery by radiation in plasmachemical reactors containing porous plates [29]. In [30] a scheme of heat recovery by radiation inside a system consisting of two highly porous plates is discussed. There, it is suggested that radiative heat transfer be employed for heating of a porous plate through which air leaks, thus causing its heating as a result of heat transfer between a gas and the skeleton of the porous plate. This makes it possible to obtain a temperature higher than the adiabatic one.
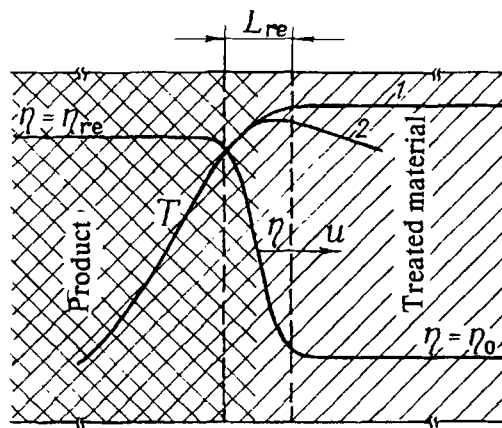
Fig. 3. Schematic of filtration combustion (η is the extent of transformation of the treated material; L_{re} is the reaction-zone width; u is the linear velocity of the reaction wave; T is the temperature): 1) without heat losses, 2) with allowance for heat losses.

8. FILTRATION COMBUSTION

By filtration combustion of porous condensed systems is understood propagation, in a porous body, of the exothermic reaction of this body (or a component of it) with the active component of a gaseous mixture (consisting of an oxidant and an inert gas) filtered through it. Though this process has long been used (sintering and agglomeration of ores), recently it has found a number of new applications.

The essence of this process lies in the fact that at some site of the reaction zone of a technological apparatus aprimary thermal wave is initiated by an "external" energy source that, transforming the initial substances into the end product, is maintained owing to the heat of the chemical reaction. Here, the reaction zone moves in the wake of the thermal wave (Fig. 3).

The method of thermal waves is used in SHS technology, i.e., in self-propagating high-temperature synthesis of high-melting and other inorganic materials [31]. In this case, gas filtration to the reaction zone is a natural event, and the process itself is self-regulating.

A combustion regime involving forced filtration of a gaseous mixture makes it possible to regulate the temperature and other characteristics of the reaction wave. In [32], an analytical theory of filtration combustion involving forced filtration of a gas is developed. This method allows the appropriate technological process to be conducted at an optimum temperature with the desired extent of transformation (oxidation) of the treated material. Filtration combustion finds diverse application in technology, e.g., agglomeration of ores [33], synthesis of materials [34], sanitary purification of industrial exhaust gases with direct combustion of toxic wastes [35], and adsorptive-catalytic processes with adsorbent recovery in a wave of filtration combustion [36].

The process discussed has another more important field of application, namely, power engineering. The case in point is combustion of gaseous fuel in burners with porous nozzles or in a layer of a porous fire-resistant material for the purpose of increasing the efficiency of the process. In [37, 38], investigations have been conducted aimed at reaching superadiabatic temperatures in a system of two porous plates upon gaseous-fuel combustion. A combustible gas with a rather low heat content flows into a porous medium, where it regularly reverses its direction. The enthalpy of the exhaust gas is regenerated efficiently by a system of recirculating flows, thus increasing the enthalpy of the combustible gas in the porous medium and, as a result, accumulating heat, i.e., a superadiabatic temperature is reached by recovery of the combustion products of the gaseous fuel, with the thermal energy being returned to the reaction zone upon changing the direction of pumping. These and other aspects of combustion in porous bodies were discussed at the International Workshop held in Minsk (Republic of Belarus) in 1997 [39].

9. HEAT EXCHANGERS WITH POROUS COATS

Heat exchangers with porous coats are in wide use in technology. Such porous inserts allow considerable (severalfold) enhancement of heat transfer from a heating surface to a liquid flowing in a porous layer [40]. Porous coats are also used for intensification of heat transfer under boiling conditions since in these cases the heat transfer crisis can occur at higher rates of heat flow. However, the influence of a porous coat on the characteristics of the boiling transfer crisis is rather complicated and depends on the coat properties and the flow structure [41].

In [42], heat transfer in filtration flow of liquids in pipes is investigated. Here, in particular, it is shown that two ranges of parameters are distinguished: at low Reynolds numbers Re_d heat transfer depends not only on Re_d but also on the ratio of the diameters of the tube and the spherical particles of the packing, while at high Re_d values it is completely determined by Re_d. In the latter case, all experimental data are grouped around the dependence [42]

$$Nu = 0.4 \, Re_d^{0.67} \, Pr^{0.4} .$$

Heat-pipe heat exchangers also find wide application [43]. Recently, heat exchangers with sorption by solids have being developed. In [44], the authors describe and analyze a design of a heat pump with sorption on a solid where heat is recovered by a heat pipe in the presence of a condensing (evaporating) coolant. Such devices are an alternative to compression systems.

10. DEFORMABLE POROUS MEDIA. HEAT AND MASS TRANSFER IN SOIL

In a rigorous account of elastic deformations of a porous skeleton, use is made of a generalized Hooke law, that is a relationship between fictitious stresses (caused by transfer of forces via contacts between grains of the solid phase), the pressure in the pores, and deformations of the solid phase [1, 45]. In this case, the tensor of macrodeformations of the solid phase is equal to the sum of the tensor determining the contribution of microdeformations of the grain material and the tensor of fictitious (effective) deformations due to displacements of grains relative to each other [1].

In [1] a complete system of filtration equations for a saturated porous medium with elastic deformations of the solid phase is given. In the system, the equations of conservation of mass and momentum of the phases and the energy equation are closed by the equations of state of the phases, by specifying the parameters of interphase interaction, and by the relationship between the fictitious stresses and deformations (in accordance with the generalized Hooke law) and the equation for porosity variation.

Soils consist of three phases, namely, solid, liquid (soil solution), and gas (air). The solid phase is a source of nutrient substances, the liquid phase provides transfer of the nutrients to the root system of plants, while the gas phase is responsible for gas transfer in the soil [46, 47].

The moisture mobility in soil depends on the structure of the latter, electrokinematic phenomena at the phase boundaries, and the rheological properties of the material.

Depending on the ratio of the phases, soils as rheological systems can exist in different states (elastic, viscoelastic, etc.). Here, it is assumed that the evolution of deformation with time and with a change in the phases occurs under the action of capillary forces [46]. If the stresses developing in the porous-body skeleton due to the capillary forces do not exceed the limiting shear stress, the deformation is elastic. Then a certain p_σ is responsible for the corresponding deformation (shrinkage) of the material. With decrease of the liquid phase, the capillary pressure p_σ increases and the volume of the skeleton decreases, while with increase in the amount of liquid p_σ decreases and the porous body swells. If the stresses exceed the limiting shear stress, plastic deformation develops [46].

The monograph [48] is devoted mainly to mathematical models of heat and mass transfer in soil. In particular, the problems of combined heat and moisture transfer in soils are investigated using a system of A. V. Luikov equations. Heat and moisture transfer in a soil–air system is investigated by the method of conjugate problems. The influence of the plant cover on the thermal and water regimes of the soil is also investigated. It is

important that statistical methods are used to analyze the temperature regime of the soil with allowance for daily and fractional daily fluctuations of agrometeorological factors. A separate chapter of the monograph [47] is devoted to methods and devices for thermophysical measurements in soil (measurements of temperature, thermal conductivity, heat flux).

An important branch of the thermophysics of soils is heat and mass transfer in frozen rock. As is emphasized in [49], in these cases the processes of transfer of heat, moisture, and dissolved substances and deformation of the structure of the material are closely interrelated.

In [50], results of experimental studies of transfer processes in the interaction of frozen rock and salt solution are reported. It is shown that in this case a whole complex of physicochemical and structurization processes accompanied by transfer of heat, moisture, and chemical components exists. The work [51] is devoted to mathematical modeling of results of anthropogenic contamination of frozen ground (in the case of rupture of pipelines, sewage systems, etc.). Here, three models are discussed: the first is based on the Stefan problem, the second allows for the water−ice phase transition in a temperature range, and in the third model it is assumed that the phase transition is accomplished with allowance for the concentration of the pore solution.

It should be noted that in [52] efficient algorithms for solving a number of multifront problems of the Stefan type that describe heat and mass transfer with phase changes in rocks of various kinds are given, including problems of freezing with allowance for moisture migration.

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